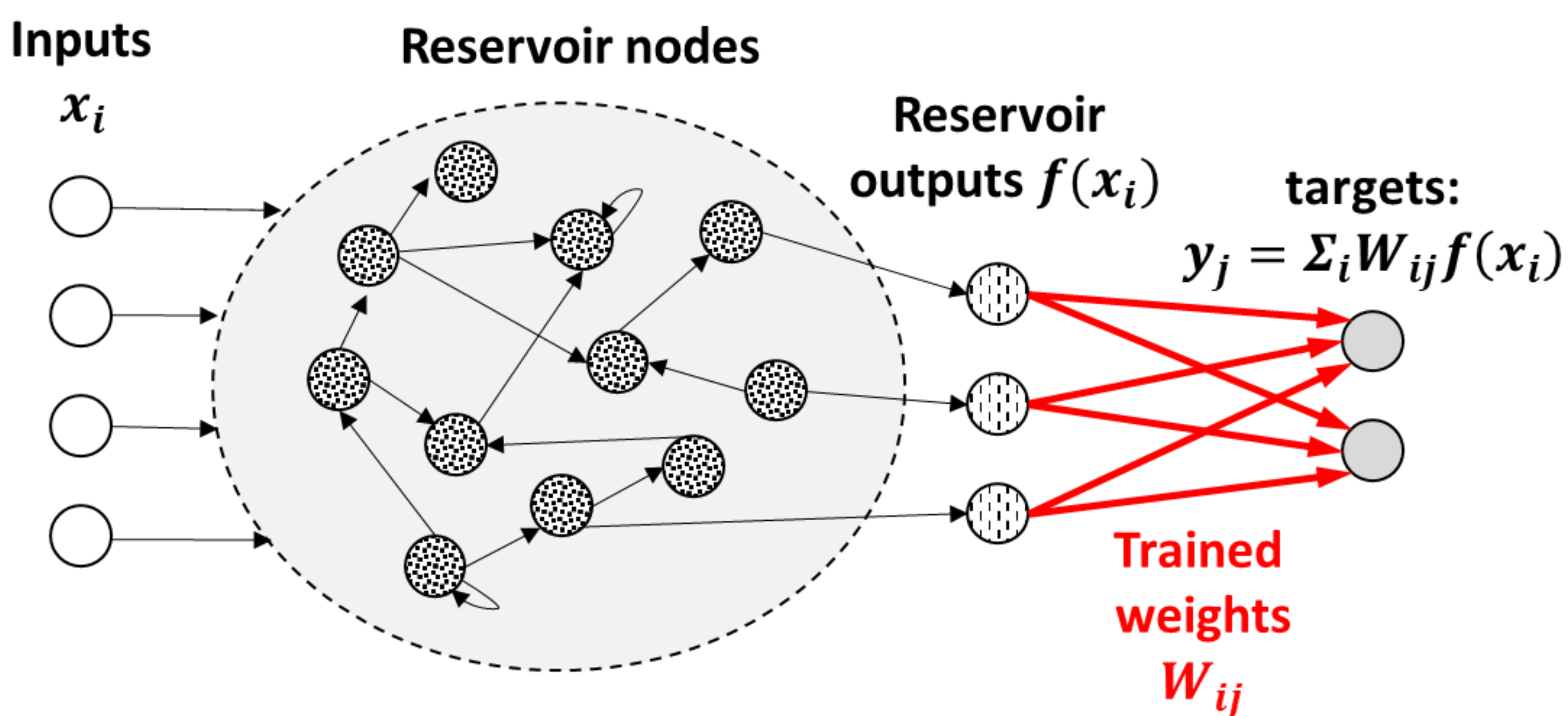


RESERVOIR COMPUTING (RC)

Reservoir computing (RC) is a computational framework derived from recurrent neural networks, where only the output layer is trained while the internal dynamics remain fixed. In classical reservoir computing, multiple input nodes are typically used, with each feature of the input data mapped to a separate node.

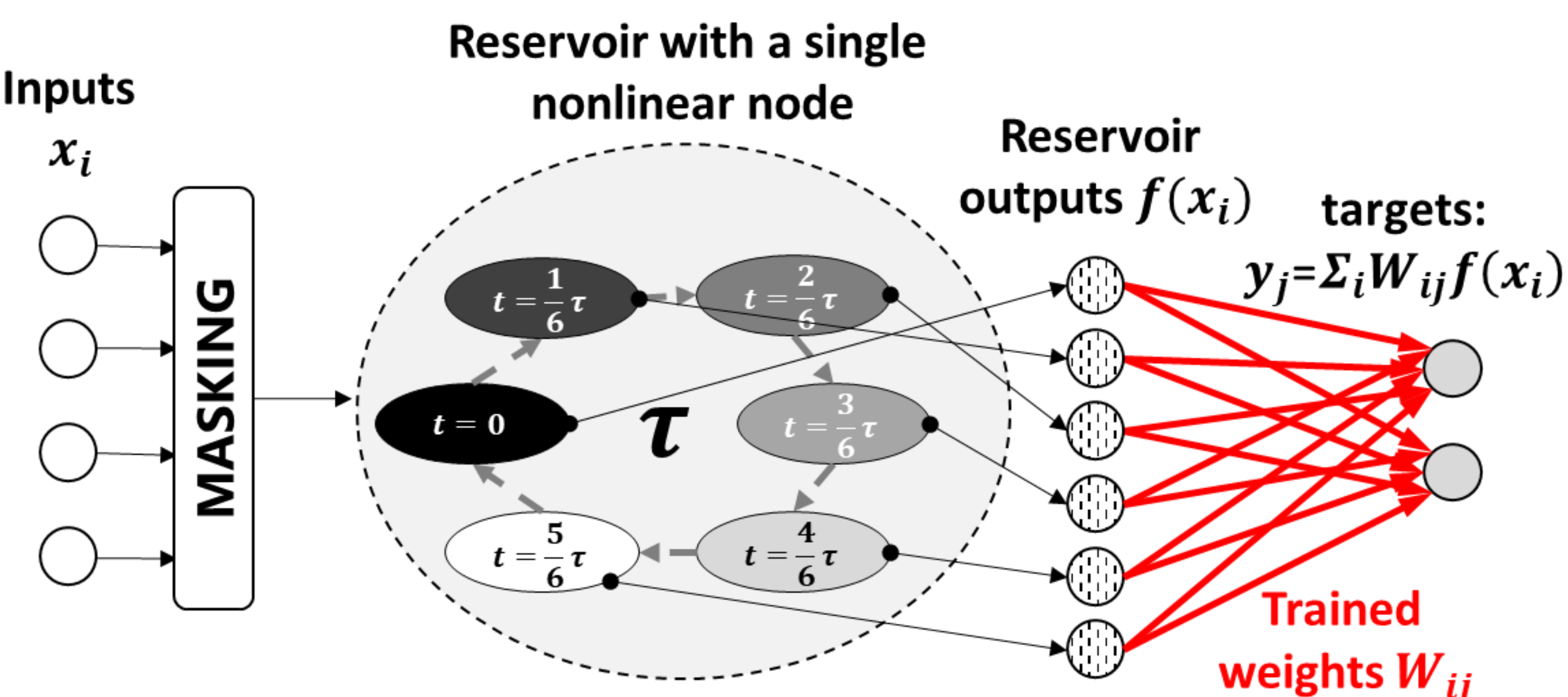
RC excels in processing temporal data, making it effective for time series analysis and prediction in areas like financial forecasting and weather modeling. Its ability to handle complex nonlinear systems also leads to successful applications in natural language processing for tasks such as speech recognition and in robotics for sensorimotor control.



The black connections in the physical reservoir (gray area) as well as the input weights (black arrows on the left side) are fixed and only the output weights W_{ij} (the red connections) are trained.

TIME-MULTIPLEXED RESERVOIR COMPUTING (TMRC)

Time-multiplexed reservoir computing utilizes delay-based systems to efficiently process information using a single dynamical node [1-2]. This method encodes input signals into a temporal mask, which is then scaled and time-multiplexed into a single input stream. Multiplying the input signal with the temporal mask signal enhances the system's performance by introducing complexity and diversity into the input data. The nonlinear dynamics of the reservoir, often implemented using optical devices, enable efficient realization of complex tasks such as time-series prediction and classification [3]. This approach has also been explored in magnonics [4].

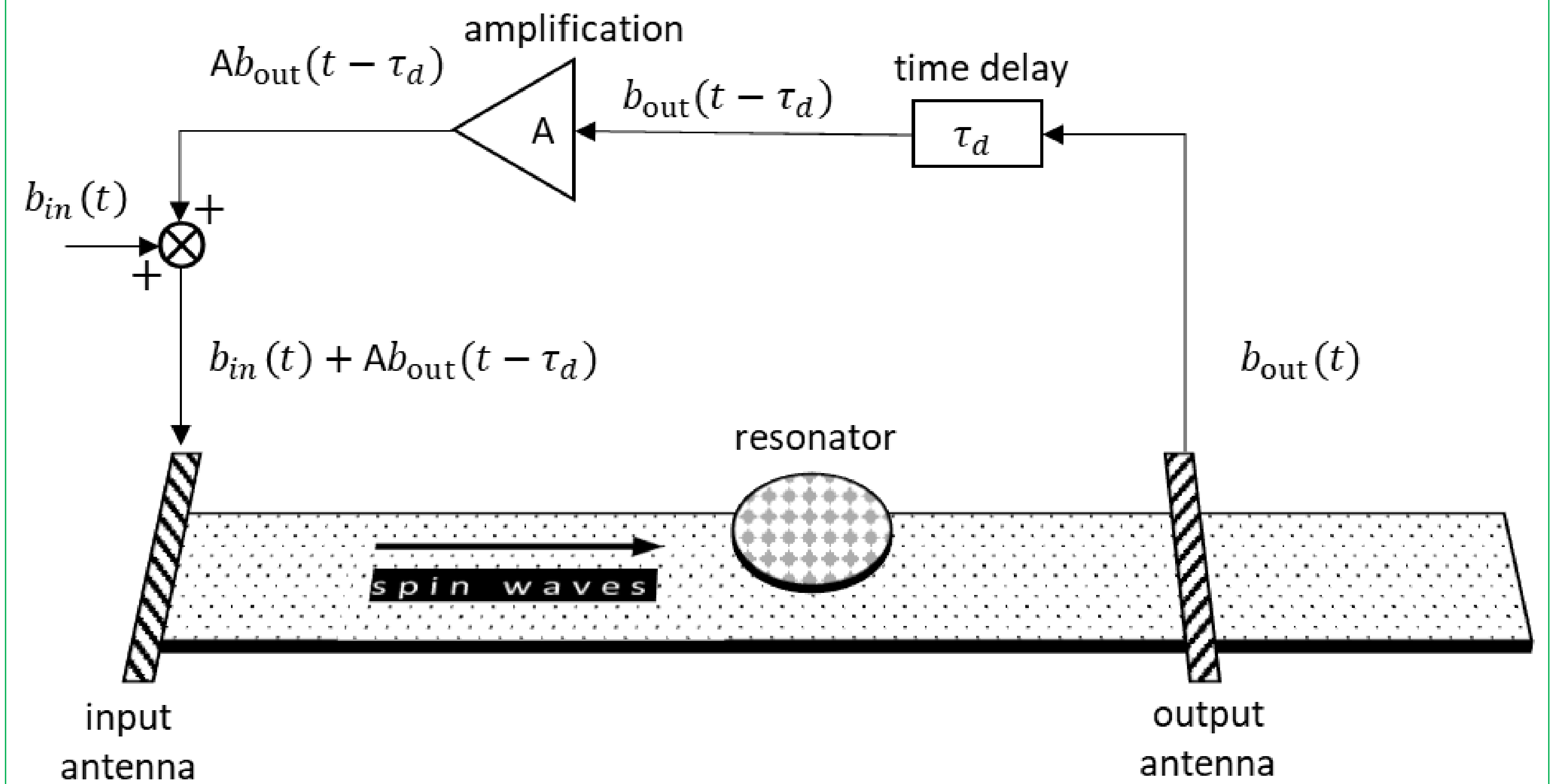


REFERENCES

- [1] L. Appeltant et al. Nat. Comm. 2, 468 (2011).
- [2] L. Appeltant et al. Sci. Rep. 4, 3629 (2014)
- [3] L. Larger et al. Phys. Rev. X, 7, 011015, (2017).
- [4] S. Watt, M. Kostylev, A. Ustinov, and B. Kalinikos. Phys. Rev. Appl. 15, 064060 (2021).

Acknowledgments: This project has received funding from the European Union's Horizon Europe research and innovation program under Grant Agreement No. 101070347-MANNGA. Yet, views and opinions expressed are those of the authors only and do not necessarily reflect those of the EU, and the EU cannot be held responsible for them.

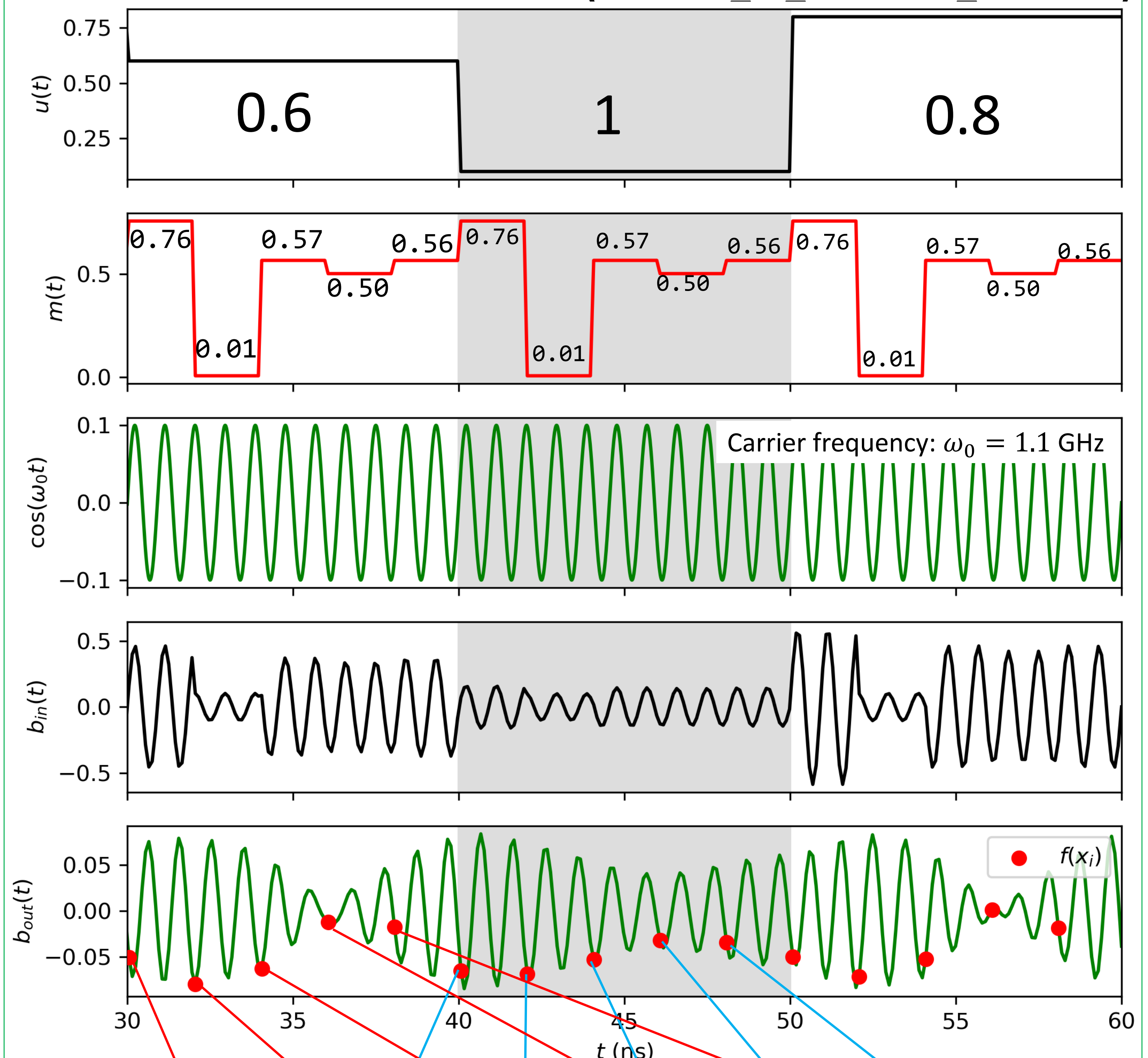
TMRC IN A SYSTEM WITH MAGNONIC RESONATOR



$$b_{in}(t) = b_0 e^{-i\omega_0 t} (1 + k_{mod} m(t) u(t))$$

$m(t) = m(t + n\tau)$ - mask signal, $u(t)$ - input stream

Example of time-multiplexed reservoir computer one nonlinear node and with 5 virtual neurons (number_of_reservoir_nodes=5)



$$y_0 = W_0 b_{out,0} + W_1 b_{out,1} + W_2 b_{out,2} + W_3 b_{out,3} + W_4 b_{out,4}$$

$$y_1 = W_0 b_{out,0} + W_1 b_{out,1} + W_2 b_{out,2} + W_3 b_{out,3} + W_4 b_{out,4}$$

The weights W_i are learned using a supervised learning to map the reservoir states to the desired target output Y_{target} . E.g., using the ridge regression, the **output weights matrix** $\mathbf{W}_{out} = [W_0, W_1, W_2, W_3, W_4]$ can be estimated:

$$\mathbf{W}_{out} = \mathbf{Y}_{target} \mathbf{B}_{out}^T (\mathbf{B}_{out} \mathbf{B}_{out}^T + \lambda \mathbf{I})^{-1}$$

- \mathbf{B}_{out} - the **reservoir states matrix**. Its dimensions are (number_of_samples \times number_of_reservoir_nodes). Each row of this matrix represents the state of all the reservoir nodes at a given time step (for a particular input sample).
- \mathbf{Y}_{target} - **target values matrix**. Its dimensions are (number_of_samples \times number_of_outputs). Each row of this matrix contains the desired output values for the corresponding input sample.
- \mathbf{I} - the identity matrix with dimensions (number_of_reservoir_nodes \times number_of_reservoir_nodes)
- λ - the ridge parameter (to prevent overfitting), $\lambda = 0$ for ordinary least squares.